Topic Test: OxfordAQA International AS Mathematics

Pure Mathematics PSM1

Name: ____________________

Class: ____________________

Date: ____________________

Time: **72 minutes**

Marks: **60 marks**

Comments:

1 A circle with centre $C(-3, 2)$ has equation

$$x^2 + y^2 + 6x - 4y = 12$$

(a) Find the y-coordinates of the points where the circle crosses the y-axis.

(3)

(b) Find the radius of the circle.

(3)

(c) The point $P(2, 5)$ lies outside the circle.

(i) Find the length of CP, giving your answer in the form $\sqrt{n}$, where n is an integer.

(2)

(ii) The point Q lies on the circle so that PQ is a tangent to the circle. Find the length of PQ.

(2)

(Total 10 marks)

2 A circle has centre C $(-3, 1)$ and radius $\sqrt{13}$.

(a) (i) Express the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = k$$

(2)

(ii) Hence find the equation of the circle in the form

$$x^2 + y^2 + mx + ny + p = 0$$

where m, n and p are integers.

(3)

(b) The circle cuts the y-axis at the points A and B. Find the distance AB.

(3)

(c) (i) Verify that the point D $(-5, -2)$ lies on the circle.

(1)

(ii) Find the gradient of CD.

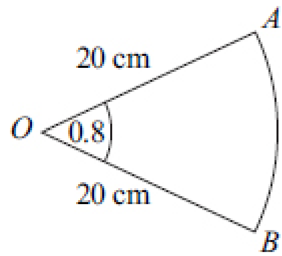
(2)

(iii) Hence find an equation of the tangent to the circle at the point D.

(2)

(Total 13 marks)

3 The diagram shows a sector OAB of a circle with centre O.



The radius of the circle is 20 cm and the angle AOB = 0.8 radians.

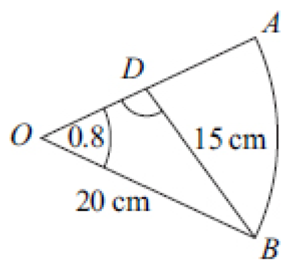
(a) Find the length of the arc AB.

(2)

(b) Find the area of the sector OAB.

(2)

(c) A line from B meets the radius OA at the point D, as shown in the diagram below.



The length of BD is 15 cm. Find the size of the **obtuse** angle ODB, in **radians**, giving your answer to three significant figures.

(4)

(Total 8 marks)

4 (a) Write down the two solutions of the equation $\tan(x + 30°) = \tan 79°$ in the interval $0° \leq x \leq 360°$.

(2)

(b) Describe a single geometrical transformation that maps the graph of $y = \tan x$ onto the graph of $y = \tan(x + 30°)$.

(2)

(c) (i) Given that $5 + \sin^2 \theta = (5 + 3\cos\theta)\cos\theta$, show that $\cos\theta = \frac{3}{4}$.

(5)

(ii) **Hence** solve the equation $5 + \sin^2 2x = (5 + 3\cos 2x)\cos 2x$ in the interval $0 < x < 2\pi$, giving your values of x in radians to three significant figures.

(3)

(Total 12 marks)

5 (a) Given that $\log_a b = c$, express b in terms of a and c.

(1)

(b) By forming a quadratic equation, show that there is only one value of x which satisfies the equation $2 \log_2(x + 7) - \log_2(x + 5) = 3$.

(6)

(Total 7 marks)

6 (a) Sketch the graph of $y = \frac{1}{2^x}$, indicating the value of the intercept on the y-axis.

(2)

(b) Use logarithms to solve the equation $\frac{1}{2^x} = \frac{5}{4}$, giving your answer to three significant figures.

(3)

(c) Given that

$$\log_a(b^2) + 3 \log_a y = 3 + 2 \log_a \left(\frac{y}{a}\right)$$

express y in terms of a and b.

Give your answer in a form not involving logarithms.

(5)

(Total 10 marks)

Mark schemes

1 (a) $x = 0 \Rightarrow y^2 - 4y - 12 = 0)$

sub x = 0 & correct quadratic in y

or $(y - 2)^2 = 16$ *or* $(y - 2)^2 - 16 = 0$

M1

$(y - 6)(y + 2) \quad (= 0)$

correct factors or formula as far as $\frac{4 \pm \sqrt{64}}{2}$

or $y - 2 = \pm\sqrt{16}$

A1

$\Rightarrow y = -2, 6$

condone (0, − 2) & (0, 6)

A1

3

(b) $(x + 3)^2 - 9 + (y - 2)^2 - 4 \ (=12)$

correct *sum of square terms* ***and*** *attempt to complete squares (or multiply out) PI by next line*

M1

$(r^2 =)\ 9 + 4 + 12$

$(r^2 =)25$ seen on RHS

A1

$(\Rightarrow r =)\ 5$

$r = \sqrt{25}$ or $r = \pm 5$ scores A0

A1

3

(c) (i) $(CP^2 =) \quad (2 -- 3)^2 + (5 - 2)^2$

condone one sign slip within one bracket

M1

$\Rightarrow (CP =) \quad \sqrt{34}$

n = 34

A1

2

(ii) $PQ^2 = CP^2 - r^2 = 34 - 25$

Pythagoras used correctly with values FT "their" r and CP

M1

$(\Rightarrow PQ =)$ 3

A1

2

[10]

2 (a) (i) $(x + 3)^2 + (y - 1)^2$

condone $(x - -3)^2$

B1

= 13

condone $(\sqrt{13})^2$

B1

2

(ii) $x^2 + 6x + 9 + y^2 - 2y + 1$

attempt to multiply out both of 'their' brackets;
must have x and y terms

M1

$x^2 + y^2 + 6x - 2y$

both m = 6 and n = –2

A1

– 3 = 0

All correct, p = –3 and ... = 0

A1

3

(b) $x = 0 \Rightarrow y^2 - 2y - 3 = 0 \Rightarrow (y - 3)(y + 1) = 0$

putting x = 0 PI
and attempt to solve or factorise

M1

$y = 3$, $y = -1$

A1

$\Rightarrow$ Distance $AB = 3 + 1 = 4$

OR *Pythagoras* $d^2 = 13 - 3^2$ *M1*

$d = 2$ *A1*

distance = 2 × 2 = 4 *A1*

A1cso

3

(c) (i) $(-5+3)^2 + (-2-1)^2 = 4 + 9 = 13$

Substitution $x = -5$, $y = -2$ into any correct circle equation

$\Rightarrow$ D lies on circle

convincing verification ***plus statement***

B1

1

(ii) grad $CD = \dfrac{1+2}{-3+5}$

condone one sign slip

M1

$= \dfrac{3}{2}$ (or 1.5)

not $\dfrac{-3}{-2}$

A1

2

(iii) Perpendicular gradient $= -\dfrac{2}{3}$

ft their grad CD or $m_1 m_2 = -1$ stated

M1

Tangent has equation $y + 2 = -\dfrac{2}{3}(x+5)$

any form of correct equation eg 2x + 3y + 16 = 0

$y = -\dfrac{2}{3}x + c,\ c = -\dfrac{16}{3}$

A1

2

[13]

3 (a) {Arc =} $r\theta = 20 \times 0.8$

$r\theta$ seen in (a) or used for the arc length

M1

… = 16 (cm)

A1

2

(b) {Area of sector =} $\frac{1}{2}r^2\theta = \frac{1}{2} \times 20^2 \times 0.8$

$\frac{1}{2}r^2\theta$ OE seen in (b) or used for the area

M1

… = 160 (cm^2)

A1

2

(c) {Let D = angle ODB} $\frac{20}{\sin D} = \frac{15}{\sin 0.8}$

Sine rule, ACF with sin D being the only unknown
PI by next line

M1

$\sin D = \frac{20 \times \sin 0.8}{15} \left\{= \frac{14.3(471...)}{15}\right\}$

$\left\{= \frac{20}{20.9(10...)}\right\}$ = 0.956(474…)

Acute ‘D = 1.27(467…)

Correct rearrangement to ‘sin D= …’ or to ‘ D = sin^{-1} (…)’
OE. PI by at least 3sf correct value 1.27(467…) radians
or 73(.033)° for acute angle or PI by at least 3sf
value 1.86(692…) rounded or truncated for D

m1

$D = \pi$ − Acute ‘D in rads

<u>Dep on previous 2 marks</u> being awarded. PI by correct ft evaluation
of π − c’s acute D to at least 3 sf value or
seeing 1.86(692…), rounded or truncated, for D

m1

{Angle ODB} = 1.87 {to 3sf}

Condone > 3sf.

A1

4

[8]

4 (a) $x + 30° = 79°$, $x + 30° = 180° + 79°$

$x = 49°$

49 as the only solution in the interval $0° \le x \le 90°$

B1

$x = 229°$

AWRT 229. Not given if any other soln. in the interval $90° \le x \le 360°$.

Ignore anything outside $0° \le x \le 360°$

B1

2

(b) Translation;

Accept 'translat…' as equivalent.
[T or Tr is NOT sufficient]

B1

$$\begin{bmatrix} -30° \\ 0 \end{bmatrix}$$

*OE Accept **full** equivalent to vector in words provided linked to 'translation / move / shift' and **correct** direction. (0 / 2 if >1 transformation).*

B1

2

(c) (i) $5 + \sin^2\theta = (5 + 3\cos\theta)\cos\theta$

$\Rightarrow 5 + \sin^2\theta = 5\cos\theta + 3\cos^2\theta$

Correct RHS.

B1

$5 + 1 - \cos^2\theta = 5\cos\theta + 3\cos^2\theta$

$\sin^2\theta = 1 - \cos^2\theta$ used to get a quadratic in cos.

M1

$6 = 5\cos\theta + 4\cos^2\theta$ or $4\cos^2\theta + 5\cos\theta - 6\ (= 0)$

ACF with like terms collected.

A1

$\Rightarrow \quad (4\cos\theta - 3)(\cos\theta + 2) \quad (= 0)$

***Correct** quadratic and $(4c \pm 3)(c \pm 2)$ or by formula OE PI by $\pm$ 'correct' 2 values for cos θ.*

m1

Since $\cos\theta \neq -2$, $\cos\theta = \frac{3}{4}$

CSO AG. Must show that the 'soln' cos $\theta = -2$ has been considered and rejected

A1

5

(ii) $5 + \sin^2 2x = (5 + 3\cos 2x)\cos 2x \Rightarrow \cos 2x = \frac{3}{4}$

*Using (c)(i) to reach cos2x = ¾ **or** finding at least 3 solutions of cos θ = ¾ **<u>and</u>** dividing them by 2.*

M1

$2x = 0.722(7...), 2\pi - 0.722(7...),$
$2\pi + 0.722(7...), \quad 4\pi - 0.722(7...),$

Valid method to find all four 'positions' of solutions.

m1

$x = 0.361, 2.78, 3.50, 5.92$

CAO Must be these four 3sf values but ignore any values outside the interval $0 < x < 2\pi$.

A1

3

[12]

5 (a) $b = a^c$

B1

1

(b) $2\log_2(x + 7) - \log_2(x + 5) = 3$
$\log_2(x + 7)^2 - \log_2(x + 5) = 3$

A law of logs used correctly on a correct expression.

M1

$\log_2 \frac{(x+7)^2}{x+5} = 3$

A further correct use of law of logs on a correct expression.

M1

$= 3\log_2 2 = \log_2 2^3$

$\Rightarrow \frac{(x+7)^2}{x+5} = 2^3$

$3 = 3\log_2 2$ or $3 = \log_2 2^3$ ($= \log_2 8$) seen
or
eg log f(x) = 3 $\Rightarrow$ f(x) = 2^3 (= 8) OE

B1

$\Rightarrow (x+7)^2 = 8(x+5)$

Correct equation having eliminated logs and fractions

A1

$\Rightarrow x^2 + 14x + 49 = 8x + 40$
$\Rightarrow x^2 + 6x + 9$ (= 0)

A1

Since $6^2 - 4(1)(9) = 0$, (there is only) one value of x
(which satisfies the given equation).

OE

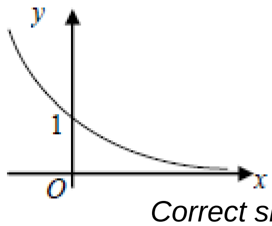
CSO Need conclusion which is also correctly justified

A1

6

[7]

6 (a)



Correct shaped graph in 1st two quadrants only and indication of correct behaviour of curve for large positive and negative vals. of x. Ignore any scaling on axes.

B1

y-intercept indicated as 1 on diagram or stated as intercept = 1 or as coords (0, 1).

B1

2

(b) $\frac{1}{2^x} = \frac{5}{4} \Rightarrow 2^{-x} = \frac{5}{4}$ (or $2^x = \frac{4}{5}$ or $2^{2-x} = 5$)

Correct 'rearrangement' to eg $2^x = \frac{4}{5}$ *or* $2^{-x} = \frac{5}{4}$ *or*

$0.5^x = 1.25$ *PI or* $\log 1 - \log 2^x = \log(5/4)$ *or better*

M1

$\log 2^{-x} = \log 1.25 \Rightarrow -x\log 2 = \log 1.25$

[$\log 2^x = \log 0.8 \Rightarrow x \log 2 = \log 0.8$]

[$\log 2^{2-x} = \log 5 \Rightarrow (2 - x) \log 2 = \log 5$]

[$2^x = 0.8$, $x\log_2 0.8$]; [$0.5^x = 1.25$, $x = \log_{0.5} 1.25$]

Takes logs of both sides of eqn of form either $2^x = k$ *or* $2^{-x} = k$ *OE and uses 3rd law of logs or log to base 2 (or base ½) correctly*

M1

$x = -0.321928 \ldots$ so $x = -0.322$ (to 3sf)

Condone > 3sf [Logs must be seen to be used otherwise ***max*** *of M1M0A0]*

A1

3

(c) $\log_a b^2 + 3\log_a y = 3 + 2\log_a \left(\frac{y}{a}\right)$

$\log_a b^2 + 3\log_a y = 3 + 2\,[\log_a y - \log_a a]$

A log law used correctly; condone missing base a.

M1

$\log_a b^2 + 3\log_a y = 3 - 2\log_a a$

A different log law used correctly condone missing base a.

M1

$\log_a b^2 y = 3 - 2(1)$ $[\text{or } \log_a b^2 y + \log_a a^2 = 3]$

Either a further different log law used correctly condone missing base a or $\log_a a = 1$ a stated / used.

M1

$\Rightarrow \log_a b^2 y = 1 \quad \Rightarrow \; b^2 y = a$

log $Z = k \Rightarrow Z = a^k$ used or a correct method to eliminate logs (dep on no misapplication of any log law OE in the whole solution) Rearrangements which require only two of the above Ms toeliminate logs correctly: award the remaining M with the m mark.

M1

$\Rightarrow \quad y = ab^{-2}$

ACF of RHS

A1

5

[10]