

Please write clearly in block capitals.

Centre number

Candidate number

Surname \_\_\_\_\_

Forename(s) \_\_\_\_\_

Candidate signature \_\_\_\_\_

I declare this is my own work.

# INTERNATIONAL A-LEVEL FURTHER MATHEMATICS

(9665/FM04) Unit FS2 Statistics

Monday 18 January 2021 07:00 GMT Time allowed: 1 hour 30 minutes

### Materials

- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

### Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

### Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80

### Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
<b>TOTAL</b>	



Answer **all** questions in the spaces provided.

- 1** A company believes that use of its revision app changes the chance of a candidate passing a particular mathematics examination.

The results of 200 candidates taking the mathematics examination were collected.

Some of these candidates used the app and others did not.

The table below shows the observed frequencies  $O_i$  from the collected data and the expected frequencies  $E_i$  necessary for a  $\chi^2$  test.

	Pass		Fail	
	$O_i$	$E_i$	$O_i$	$E_i$
App used	72	66	28	34
App <b>not</b> used	60	66	40	34

The company conducts a  $\chi^2$  test using a 10% level of significance.

- 1 (a)** State the hypotheses for this test.

[1 mark]

---



---



---

- 1 (b)** Show that the value of the test statistic  $\chi^2$  is 2.696 correct to four significant figures.

[2 marks]

---



---



---



---



---



---



---





**There are no questions on this page**

*Do not write  
outside the  
box*

**DO NOT WRITE ON THIS PAGE  
ANSWER IN THE SPACES PROVIDED**









- 4 The random variable  $X$  has a distribution with unknown mean  $\mu$  and unknown variance  $\sigma^2$

A random sample of size  $n$ , denoted by  $X_1, X_2, \dots, X_n$  is used to determine two statistics

$$T = \sum_{k=1}^n X_k \quad \text{and} \quad V = \left( \frac{1}{n} \sum_{k=1}^n X_k^2 \right) - \frac{T^2}{n^2}$$

- 4 (a) Explain why  $T$  and  $V$  are both statistics.

[2 marks]

---

---

---

---

---

- 4 (b) Show that  $T$  is not an unbiased estimator.

[2 marks]

---

---

---

---

---



4 (c) (i) Show that  $E(X_k^2) = \sigma^2 + \mu^2$

[1 mark]

---



---



---



---

4 (c) (ii) Show that  $E(T^2) = n\sigma^2 + n^2\mu^2$

[2 marks]

---



---



---



---



---



---



---



---

4 (d) Use your results in **part (b)** and **part (c)** to show that  $\frac{nV}{n-1}$  is an unbiased estimator of  $\sigma^2$

[3 marks]

---



---



---



---



---



---



---



---



---



---







**6** A particular model of a mobile phone has a battery lifetime  $X$  hours with mean  $\mu$  and variance  $\sigma^2$

**6 (a)** A random sample of size  $n$  of the phones is taken and the mean  $\bar{X}$  is calculated.

**6 (a) (i)** Write down expressions for the mean and variance of  $\bar{X}$  in terms of  $\mu$ ,  $\sigma$  and  $n$  **[2 marks]**

---



---

Mean \_\_\_\_\_ Variance \_\_\_\_\_

**6 (a) (ii)** Hence show that  $\bar{X}$  is an unbiased and consistent estimator of  $\mu$  **[2 marks]**

---



---



---



---

**6 (b)** Shop  $A$  and shop  $B$  both sell this model of mobile phone.

Shop  $A$  surveys a random sample of 40 customers and calculates the mean  $\bar{X}_A$  as an estimate of  $\mu$

Shop  $B$  surveys a different random sample of 60 customers and calculates the mean  $\bar{X}_B$  as an estimate of  $\mu$

Show that the relative efficiency of  $\bar{X}_B$  with respect to  $\bar{X}_A$  is 1.5

**[2 marks]**

---



---



---



---



---



---





**7** The number of platelets in the blood of males and females is being investigated.

It is assumed that the number of platelets is normally distributed with the same variance for both male and female populations.

The number of platelets, measured in thousands per microlitre, in the blood of 11 males and 9 females was recorded as

Male	261	277	201	219	349	300	321	219	277	280	310
Female	211	190	273	301	220	295	192	231	183		

**7 (a)** Using the samples, calculate a pooled estimate of the population variance.  
Give your answer to four significant figures.

**[3 marks]**

---



---



---



---



---



---



---



---

Answer \_\_\_\_\_

**7 (b)** Investigate the hypothesis that the mean number of platelets is greater for males than for females at the 5% level of significance.

**[7 marks]**

---



---



---



---



---



---



---



---







- 8 (c)** A fair six-sided die is rolled until two sixes are obtained and the total number of rolls is recorded.

The total number of rolls is calculated as the number of rolls to obtain the first six **plus** the number of further rolls to obtain the second six.

For example, if the second six appears on the 20th roll, then the total number of rolls is 20

- 8 (c) (i)** Show that the moment generating function for the total number of rolls needed is

$$\frac{1}{(6e^{-t} - 5)^2}$$

**[2 marks]**

---

---

---

---

---

---

---

---

---

---

- 8 (c) (ii)** Hence show that the mean total number of rolls needed is 12

**[2 marks]**

---

---

---

---

---

---

---

---

---

---

**Question 8 continues on the next page**

**Turn over ►**





**There are no questions printed on this page**

*Do not write  
outside the  
box*

**DO NOT WRITE ON THIS PAGE  
ANSWER IN THE SPACES PROVIDED**











**There are no questions printed on this page**

*Do not write  
outside the  
box*

**DO NOT WRITE ON THIS PAGE  
ANSWER IN THE SPACES PROVIDED**

**Copyright information**

For confidentiality purposes, all acknowledgements of third-party copyright material are published in a separate booklet. This booklet is published after each live examination series and is available for free download from [www.oxfordaqaxams.org.uk](http://www.oxfordaqaxams.org.uk).

Permission to reproduce all copyright material has been applied for. In some cases, efforts to contact copyright-holders may have been unsuccessful and Oxford International AQA Examinations will be happy to rectify any omissions of acknowledgements. If you have any queries please contact the Copyright Team.

Copyright © 2021 Oxford International AQA Examinations and its licensors. All rights reserved.

