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I declare this is my own work.

INTERNATIONAL A-LEVEL FURTHER MATHEMATICS

(9665/FM03) Unit FP2 Pure Mathematics

Time allowed: 2 hours 30 minutes

Materials

- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphic calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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7	
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9	
10	
11	
12	
13	
14	
TOTAL	



2 The matrix $\mathbf{A} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2 (a) Describe fully the single transformation represented by the matrix \mathbf{A}

[2 marks]

2 (b) For this transformation, state the line of invariant points.

[1 mark]

Answer _____

3

Turn over for the next question

Turn over ►



- 3 (a) Express $\frac{6}{(r-1)(r+1)}$ in the form $\frac{A}{r-1} + \frac{B}{r+1}$, where A and B are integers.

[2 marks]

$$\frac{6}{(r-1)(r+1)} = \underline{\hspace{10em}}$$

- 3 (b) Use the method of differences to show that

$$\sum_{r=2}^n \frac{6}{(r-1)(r+1)} = \frac{an^2 + bn + c}{2n(n+1)}$$

where a , b and c are integers.

[4 marks]



- 6 (a) A student states that vectors \mathbf{r} , \mathbf{m} and \mathbf{n} can be found such that

$$\mathbf{r} \times \mathbf{m} = \mathbf{n} \quad \text{and} \quad \mathbf{m} \cdot \mathbf{n} = 12$$

Explain why the student is **not** correct.

[2 marks]

- 6 (b) The points A , B and C have position vectors \mathbf{a} , \mathbf{b} and \mathbf{c} respectively relative to an origin O , where

$$\mathbf{a} = 2\mathbf{i} + p\mathbf{j} - \mathbf{k} \quad \mathbf{b} = -p\mathbf{i} + 4\mathbf{j} - 7\mathbf{k} \quad \mathbf{c} = -4\mathbf{i} + 2p\mathbf{j} - 9\mathbf{k}$$

and p is real.

The position vectors \mathbf{a} , \mathbf{b} and \mathbf{c} define the edges of a parallelepiped.

The volume of the parallelepiped is 17 cubic units.

Use a scalar triple product to find the four possible values of p

[6 marks]

Answer _____



8 (a) By direct expansion, or otherwise, show that

$$\begin{vmatrix} k & 2 & k-4 \\ 2k-2 & 3k-2 & 4 \\ 2k+3 & 3k & 5 \end{vmatrix} = -8k^2 + pk + q$$

where p and q are positive integers.

[2 marks]

8 (b) A system of equations is given such that

$$kx + 2y + (k - 4)z = a$$

$$(2k - 2)x + (3k - 2)y + 4z = b$$

$$(2k + 3)x + 3ky + 5z = c$$

where k , a , b and c are real constants.

8 (b) (i) Find the two values of k for which the system of equations does **not** have a unique solution.

[2 marks]



9 (a) Explain why the cubic equation

$$ax^3 + bx^2 + cx + 8 = 0$$

where a , b and c are real numbers, cannot have exactly one non-real root.

[1 mark]

9 (b) The equation

$$2z^3 + pz^2 + 4z - 6i = 0$$

where p is a constant, has roots α , β and γ

9 (b) (i) Show that

$$(\alpha\beta + 2)(\alpha\gamma + 2)(\beta\gamma + 2) = k - 3ip$$

where k is an integer.

[4 marks]



11 The plane Π_1 has equation $\mathbf{r} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} + \mu \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

The plane Π_2 has equation $\mathbf{r} \cdot \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix} = 5$

11 (a) Find an equation for the plane Π_1 in the form $\mathbf{r} \cdot \mathbf{n} = d$

[4 marks]

Answer _____

11 (b) Find the acute angle between the planes Π_1 and Π_2 giving your answer to the nearest 0.1°

[4 marks]



Answer _____

11 (c) Write down a Cartesian equation of the plane Π_2

[1 mark]

Answer _____

11 (d) Find a vector equation for the line of intersection of the planes Π_1 and Π_2 giving your answer in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$

[5 marks]

Answer _____



12 It is given that $y = \ln \left[e^{2x} (1 + \tan^2 x) \right]$

12 (a) (i) Show that $\frac{dy}{dx} = 2(1 + \tan x)$

[2 marks]

12 (a) (ii) Find $\frac{d^4y}{dx^4}$ in terms of x

[3 marks]

Answer _____



- 13 (a)** Use the definitions of $\cosh \theta$ and $\sinh \theta$ in terms of e^θ and $e^{-\theta}$ to show that

$$1 + \sinh^2 \theta = \cosh^2 \theta$$

[3 marks]

- 13 (b)** Use an integrating factor to find the general solution of the differential equation

$$\frac{dy}{dx} + \frac{x}{1+x^2}y = 2$$

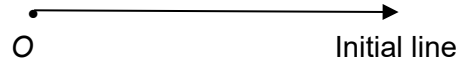
Give your answer in the form $y = f(x)$

[11 marks]



14 (b) (i) Sketch the curve C_2

[2 marks]



14 (b) (ii) By finding the polar equation of the curve C_1 , or otherwise, show that the area of the region bounded by C_1 and C_2 and the line segments PD and QE is

$$\frac{1}{2} \left(a e^{\frac{\pi}{2}} + b + c \pi \right)$$

where a , b and c are integers.

[5 marks]

END OF QUESTIONS



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