

Please write clearly in block capitals.

Centre number

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Candidate number

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Surname

Forename(s)

Candidate signature

I declare this is my own work.

INTERNATIONAL A-LEVEL FURTHER MATHEMATICS

(9665/FM03) Unit FP2 Pure Mathematics

Tuesday 11 January 2022 07:00 GMT Time allowed: 2 hours 30 minutes

Materials

- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

| For Examiner's Use | |
|--------------------|------|
| Question | Mark |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| 13 | |
| TOTAL | |



Answer **all** questions in the spaces provided.

1 (a) The matrix $\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Describe fully the **single** transformation represented by the matrix \mathbf{A}

[2 marks]

1 (b) The matrix $\mathbf{B} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

State the line of invariant points for the transformation represented by the matrix \mathbf{B}

[1 mark]

Answer _____



5 (a) Use the trigonometric identity

$$\sin A - \sin B = 2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)$$

to show that

$$\frac{1}{2} [\sin(2r+1)x - \sin(2r-1)x] = \cos 2rx \sin x$$

[1 mark]

5 (b) Hence use the method of differences to show that

$$\sum_{r=1}^n \sin^2 rx = \frac{n}{2} - \frac{\sin nx \cos(n+1)x}{2 \sin x}$$

[6 marks]



8 The plane Π_1 has vector equation

$$\mathbf{r} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 14$$

8 (a) Find the shortest distance from the origin to the plane Π_1 giving your answer in an exact form.

[2 marks]

Answer _____

8 (b) The line L has Cartesian equations

$$\frac{x-2}{3} = \frac{y+1}{2} = 2z-4$$

The line L intersects the plane Π_1 at the point P

8 (b) (i) Find the coordinates of P

[3 marks]

Answer _____



9 The matrix \mathbf{M} is defined as

$$\mathbf{M} = \begin{bmatrix} 4 & 3 & k \\ 5 & 4 & k+1 \\ 1 & 1 & 3 \end{bmatrix}$$

where k is a constant.

9 (a) Show that \mathbf{M} is a non-singular matrix.

[2 marks]

9 (b) Find \mathbf{M}^{-1} in terms of k

[5 marks]

Answer _____



- 10 (a)** Use the Maclaurin series for $\ln(1+x)$ to show that the first three non-zero terms in the Maclaurin series expansion in ascending powers of x of

$$\tanh^{-1}x \text{ are } x + \frac{x^3}{3} + \frac{x^5}{5}$$

[3 marks]

- 10 (b)** It is given that $y = \tan x$

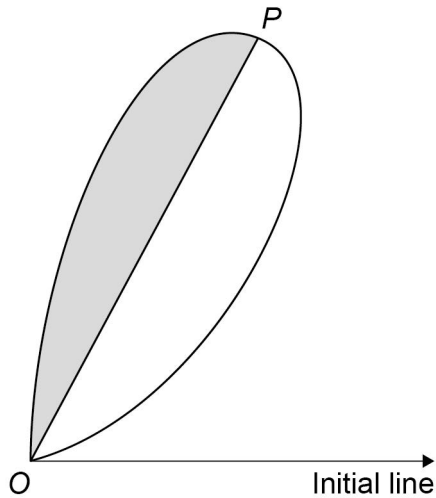
- 10 (b) (i)** Show that when $x = 0$

$$\frac{d^5 y}{dx^5} = 16$$

[4 marks]



- 12 (b)** The diagram shows a curve, the pole O , the initial line and a point P which lies on the curve.



P is the point on the curve that is furthest from the pole O

The curve has polar equation $r = 32 \sin^3 \theta \cos \theta$ where $0 \leq \theta \leq \frac{\pi}{2}$

- 12 (b) (i)** By differentiating r with respect to θ find the polar coordinates of the point P **[4 marks]**

Answer _____



- 13 (b)** It is given that the roots of the cubic equation

$$kx^3 - 36x^2 + mx - 3 = 0$$

where k and m are real constants, can be arranged as three successive terms of an arithmetic sequence with common difference d

- 13 (b) (i)** Find an expression for d^2 in terms of k

[2 marks]

Answer _____

- 13 (b) (ii)** Given that $m = 38$ find the possible values for d giving your values in an exact form.

[4 marks]

Turn over ►



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