AQA Physics A-level

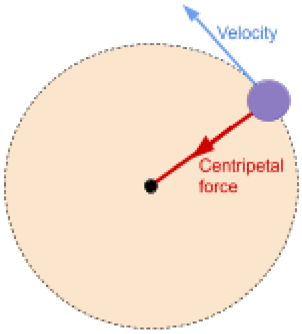
Section 6.1: Further mechanics

Notes

3.6.1 Periodic motion

3.6.1.1 - Circular motion

An object moving in a circular path at constant speed has a constantly changing velocity as velocity has both magnitude and direction, therefore the object must be **accelerating** (this is known as centripetal acceleration). We know from Newton's first law that to accelerate, an object must experience a resultant force, therefore an object moving in a circle must experience a force, this is known as the **centripetal force**, and it always acts towards the centre of the circle.

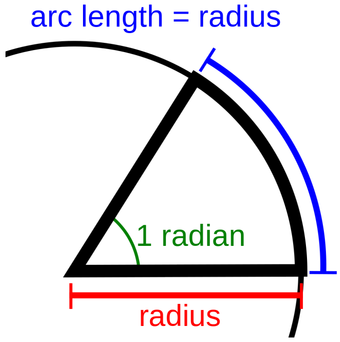


Angular speed (ω) is the angle an object moves through per unit time. It can be found by dividing the object's linear speed (v) by the radius of the circular path it is travelling in (r), or by dividing the angle in a circle (in radians) by the object's time period (T).

$$\omega = \frac{v}{r} = \frac{2\pi}{T} = 2\pi f \quad \text{as } f = \frac{1}{T}$$

Angles can be measured in units called **radians**. One radian is defined as the angle in the sector of a circle when the arc length of that sector is equal to the radius of the circle, as shown in the diagram below.

Considering a complete circle, its arc length is $2\pi r$, dividing this by r, you get 2π which is the angle in radians of a full circle. From this you can convert any angle from **degrees to radians** by multiplying by $\frac{\pi}{180}$, and from **radians to degrees** by multiplying by $\frac{180}{\pi}$.



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Centripetal acceleration (a) can be found using the formula below:

$$a = \frac{v^2}{r} = \omega^2 r$$

Using Newton's second law, $F = ma$, we can derive the formula for **centripetal force (F)** from the formula above.

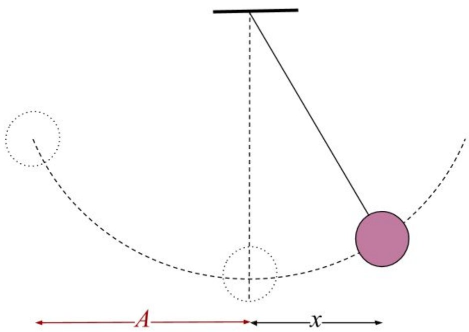
$$F = \frac{mv^2}{r} = m\omega^2 r$$

3.6.1.2 - Simple harmonic motion (SHM)

An object is experiencing **simple harmonic motion** when its acceleration is **directly proportional** to **displacement** and is in the **opposite direction**. These conditions can be shown through the equation:

$$a = -\omega^2 x$$

Where a is acceleration, ω is angular speed, x is displacement from the equilibrium position



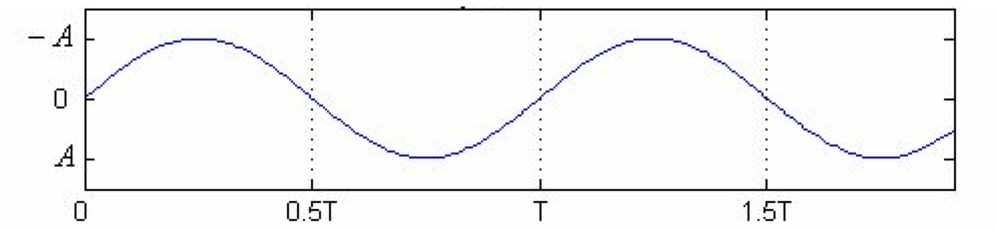
An example of a simple harmonic oscillator is the simple pendulum, as shown in the diagram above. The pendulum oscillates around a central midpoint known as the equilibrium position. Marked on the diagram by an x is the measure of **displacement**, and by an A is the **amplitude** of the oscillations, this is the maximum displacement. You could also measure the **time period (T)** of the oscillations by measuring the time taken by the pendulum to move from the equilibrium position, to the maximum displacement to the left, then to the maximum displacement to the right and back to the equilibrium position.

Using the measurements above, you could use the following formulas:

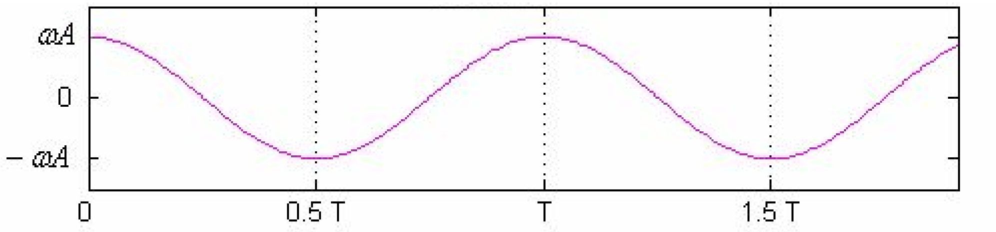
$$x = A\cos \omega t \qquad v = \pm\omega\sqrt{(A^2 - x^2)}$$ where v is velocity

You can represent the **displacement (x)**, **velocity (v)** and **acceleration (a)** of a simple harmonic oscillator on a graph. From the first equation above you can see that the **displacement-time**

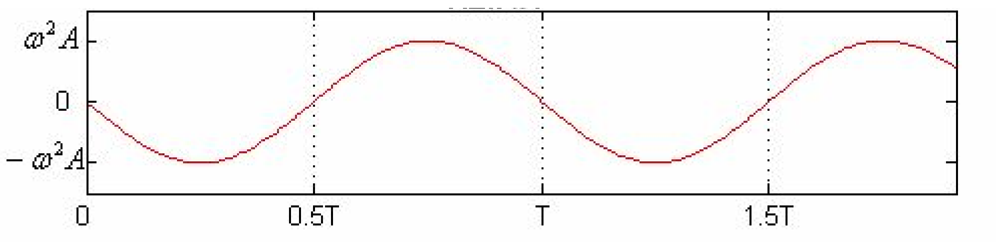
graph will follow a cosine or sine curve, with a maximum A, and minimum -A because A and **ω** are constants:



As we know that **velocity is the derivative of displacement**, we can draw a **velocity-time graph** by drawing the **gradient function of the above graph**, noting that the maximum (ωA) and minimum velocity ($-\omega A$)) occurs when x is 0, as expected from the above formula:



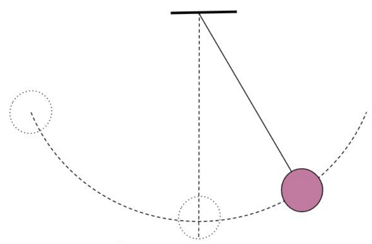
Finally, we know **acceleration is the derivative of velocity**, so we can draw an **acceleration-time graph** by drawing the **gradient function of the above graph**.



As seen above **maximum speed =** ωA and **maximum acceleration =** $\omega^2 A$.

3.6.1.3 - Simple harmonic systems

Simple harmonic systems are those which oscillate with simple harmonic motion, examples include:



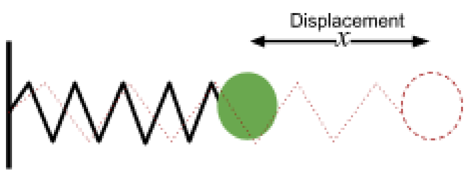
- **Simple pendulum** - A small, dense bob of mass m hangs from a string of length l , which is attached to a fixed point. When the bob is displaced by a small angle (less than 10°), and let go it will oscillate with SHM. For this type of system, you can use the following formula:

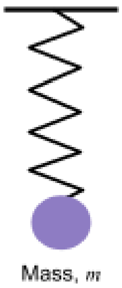
$$T = 2\pi\sqrt{\frac{l}{g}}$$

where T is time period, l is the length of the string, g is acceleration due to gravity

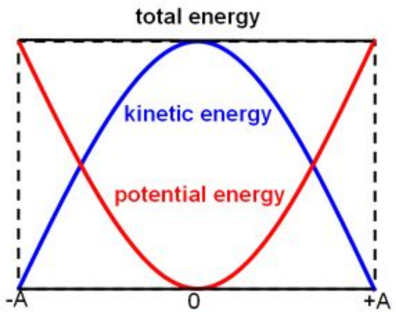
The reason the angle by which the pendulum is displaced **must be less than 10°**, is because during the derivation of the above formula a **small angle approximation** is used, and so for larger initial angles this approximation is no longer valid, and would not be a good model.
During the oscillations of a simple pendulum, its gravitational potential energy is transferred to kinetic energy and then back to gravitational potential energy and so on.

- **Mass-spring system** - There are two types of mass-spring system, where the spring is either vertical or horizontal, the only difference between these two is the type of energy which is transferred during oscillations. For the vertical system, kinetic energy is converted to both elastic and gravitational potential energy, whereas for the horizontal system, kinetic energy is converted only to elastic potential energy. For either of these types of system you can use the following formula:
$T = 2\pi\sqrt{\frac{m}{k}}$ where T is time period, m is the mass, k is the spring constant





For any simple harmonic motion system, **kinetic energy is transferred to potential energy and back as the system oscillates**, the type of potential energy depends on the system.
At the **amplitude** of its oscillations the system will have the maximum amount of potential energy, as it moves towards the equilibrium position, this potential energy is converted to kinetic energy so that at the **centre of its oscillations** the kinetic energy is at a maximum, then as the system moves away from the equilibrium again, the kinetic energy is transferred to potential energy until it is at a maximum again and this process repeats for one full oscillation. **The total energy of the system remains constant** (when air resistance is negligible, otherwise energy is lost as heat).



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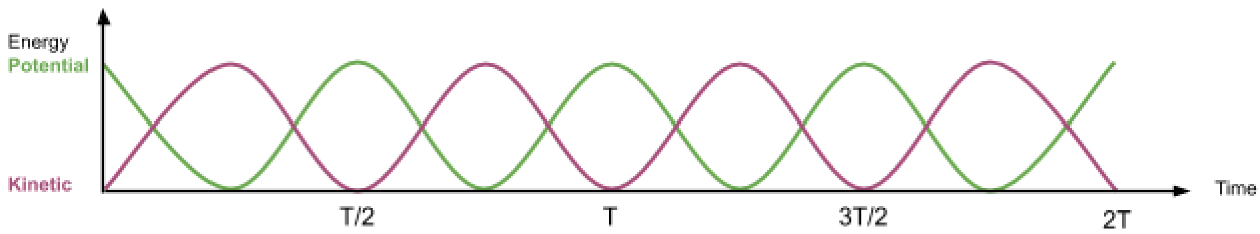
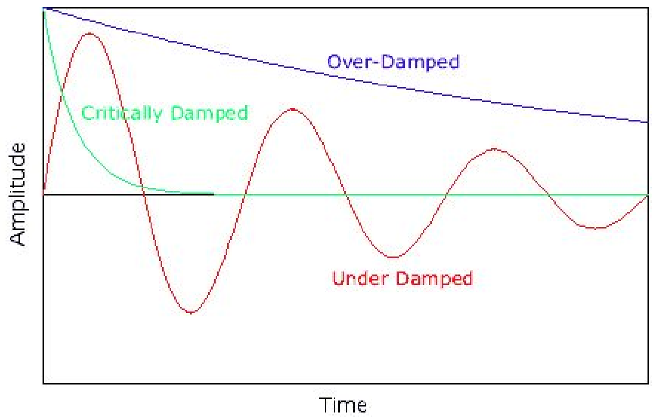
The diagram above shows the **variation of energy with displacement**, while the diagram below shows the **variation of energy with time**, for a simple harmonic system staring at its amplitude.



Damping is where the energy in an oscillating is lost to the environment, leading to reduced amplitude of oscillations. There are 3 main types of damping:

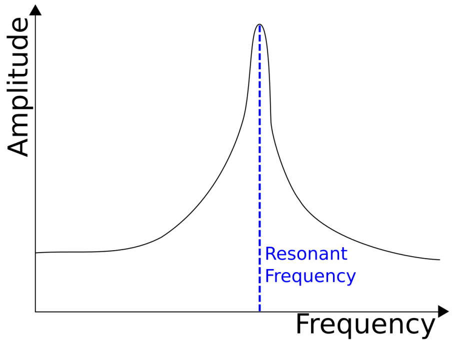
- **Light damping** - This is also known as under-damping and this is where the amplitude gradually decreases by a small amount each oscillation.
- **Critical damping** - This reduces the amplitude to zero in the shortest possible time (without oscillating).
- **Heavy damping** - This is also known as over-damping, and this is where the amplitude reduces slower than with critical damping, but also without any additional oscillations.



3.6.1.4 - Forced vibrations and resonance

Free vibrations occur when no external force is continuously acting on the system, therefore the system will oscillate at its **natural frequency**.

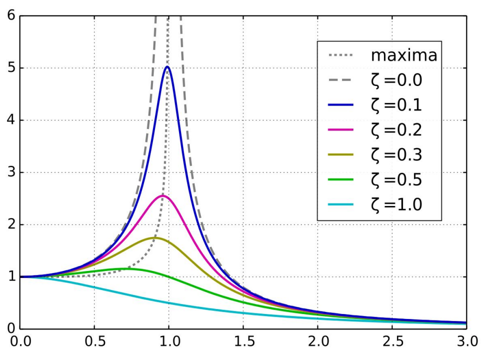
Forced vibrations are where a system experiences an **external driving force** which causes it to oscillate, the frequency of this driving force, known as driving frequency, is significant. If the **driving frequency is equal to the natural frequency of a system** (also known as the resonant frequency), then **resonance** occurs.



Resonance is where the amplitude of oscillations of a system drastically increase due to gaining an increased amount of energy from the driving force. Resonance has many applications for example:

- **Instruments** - An instrument such as a flute has a long tube in which air resonates, causing a stationary sound wave to be formed.
- **Radio** - These are tuned so that their electric circuit resonates at the same frequency as the desired broadcast frequency.
- **Swing** - If someone pushes you on a swing they are providing a driving frequency, which can cause resonance if it's equal to the resonant frequency and cause you to swing higher.

However, resonance can also have negative consequences, such as causing damage to a structure, for example a **bridge** when the people crossing it are providing a driving frequency close to the natural frequency, it will begin to oscillate violently which could be very dangerous and damage the bridge. Therefore **damping can be used to decrease the effect of resonance**, different types of damping will have different effects, **as the degree of damping increases**, the resonant frequency decreases (shifts to left on a graph), the maximum amplitude decreases and the peak of maximum amplitude becomes wider, these effects are shown in the graph below, where ζ is the damping ratio, $\zeta = 1$ represents critical damping.



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